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**Journal**

Physical Review D, 2(1)

**ISSN**

0556-2821

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**Publication Date**

1970

**DOI**

10.1103/PhysRevD.2.224

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Peer reviewed

PHYSICAL REVIEW D

VOLUME 2, NUMBER 1

1 JULY 1970

## Dips at Nonsense Wrong-Signature Points

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(Received 27 March 1970)

A model for residues of odd- and even-signature trajectories is presented which successfully accounts for the presence of dips in the differential cross sections of two-body reactions near the forward directions.

### I. INTRODUCTION

THE differential cross section for two-body or quasi-two-body final-state collisions often exhibits a dip at  $t \sim -0.6 \text{ GeV}^2$ . This dip has normally been associated with the vanishing of the residues of the  $\rho$  and  $\omega$  even-signature trajectories at the nonsense wrong-signature point, corresponding to the above  $t$  value.<sup>1</sup> However, the seemingly erratic presence or absence of this dip in different reactions has cast doubt on this explanation. We shall offer an interpretation which does rely on the vanishing of the above residues at  $t = -0.6 \text{ GeV}^2$ . Through the use of duality and the existence of exotic channels, we shall be able to determine for which reactions dips should or should not be observed.<sup>2</sup> Before

stating the results of our model, we shall indicate the assumptions which are implicit in the model.

(i) Dominant features of two-body reactions are given by the Regge-pole model. For the region of momentum transfers of interest, the trajectories considered will be the Pomeranchukon  $P$  and the quartet of vector and tensor trajectories  $\rho$ ,  $\omega$ ,  $P'$ , and  $A_2$ .

(ii) Departures from the pure Regge-pole model (with the above trajectories) are significant only in the region where contributions of the dominant trajectories vanish. These deviations may be due to cuts or lower-lying trajectories. In cross sections, these deviations will manifest themselves in their interference with the dominant terms.

(iii) The dominant non-Pomeranchukon trajectories

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<sup>1</sup> For a recent review of Regge-pole lore see G. E. Hite, *Rev. Mod. Phys.* **41**, 669 (1969).

<sup>2</sup> A previous use of duality and the existence of exotic channels

to account for these dips was presented by J. Finkelstein, *Phys. Rev. Letters* **22**, 362 (1969).

are exchange degenerate; i.e.,

$$\alpha_\rho(t) = \alpha_\omega(t) = \alpha_{P'}(t) = \alpha_{A_2}(t)$$

and  $\alpha(t) = 0$  for  $t \sim -0.6 \text{ GeV}^2$ .

(iv) We assume duality in the sense of the finite-energy sum rules. Specifically, we shall force the imaginary part of the non-Pomeranchukon Regge exchanges to vanish whenever the direct channel is exotic.<sup>3</sup> We shall consider as those meson channels with  $I=2$  or  $I=\frac{3}{2}$ , and baryon channels with  $S=+1$ . The question of whether the baryon-number-two channel is exotic will be left open at this point.

Before discussing the rules we obtain, we review briefly the experimental situation. It is summarized in Tables I and II.<sup>4</sup> In Table I we present reactions dominated by a single Regge exchange. In Table II we list reactions where more than one trajectory is present. The rules we obtain are as follows.

(i) For reactions dominated by vector trajectories a dip occurs only in the case when both initial and final mesons have nonzero isospin.

(ii) Reactions with no Pomeranchukon contributions but with  $A_2$  exchange do not exhibit any dip structure.

(iii) For reactions with a Pomeranchukon contribution, a definite statement can be made only for the cases where either the process under consideration or the line-reversed one is exotic. Our rule then states that for reactions with an exotic  $s$  channel no dip will be seen, while for those with a nonexotic  $s$  channel a moving dip will occur.

TABLE I. Dip structure in reactions dominated by one Regge trajectory.

Reaction	Dip observed	Dominant exchange
$\pi^- p \rightarrow \pi^0 n$	Yes <sup>a</sup>	$\rho$
$\pi N \rightarrow \omega N$	No <sup>b</sup>	$\rho$
$\pi N \rightarrow \rho N$	Yes <sup>c</sup>	$\omega$
$\gamma N \rightarrow \pi^+ N$	No <sup>d</sup>	$\rho$
$\gamma p \rightarrow \pi^0 p$	Yes <sup>d</sup>	$\omega$
$\gamma p \rightarrow \eta p$	No <sup>e</sup>	$\rho$
$\pi N \rightarrow \pi \Delta$	Yes <sup>f</sup>	$\rho$
$\pi N \rightarrow \omega \Delta$	No <sup>b</sup>	$\rho$
$\pi^- p \rightarrow \eta n$	No <sup>a</sup>	$A_2$
$\pi^- p \rightarrow \eta \Delta$	No <sup>a</sup>	$A_2$

<sup>a</sup> List of experimental references in R. D. Mathews, Nucl. Phys. **B11**, 339 (1969).

<sup>b</sup> L. di Lella, in Ref. 10, p. 159.

<sup>c</sup> List of experimental references in G. V. Dass and C. D. Frogatt, Nucl. Phys. **B8**, 661 (1968).

<sup>d</sup> B. Richter, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 3.

<sup>e</sup> D. Bellinger *et al.*, Phys. Rev. Letters **21**, 1205 (1968).

<sup>f</sup> G. Belletini, in Ref. d, p. 329.

<sup>3</sup> H. Harari, Phys. Rev. Letters **20**, 1395 (1968); P. G. O. Freund, *ibid.* **20**, 235 (1968).

<sup>4</sup> The first eight reactions of Table I were discussed by H. Harari, in *Proceedings of the 1969 International Symposium on Electron and Photon Interactions at High Energies, Liverpool, 1969* (unpublished).

TABLE II. Dip structure in reactions dominated by several Regge trajectories.

Reaction	Dip observed	Dominant exchange
$K^- p \rightarrow K^0 n$	No <sup>a</sup>	$\rho + A_2$
$K^+ p \rightarrow K^0 \Delta^{++}$	No <sup>a</sup>	$\rho + A_2$
$K^+ p \rightarrow K^+ p$	No <sup>b</sup>	$P + P' - \omega$
$p p \rightarrow p p$	No <sup>b</sup>	$P + P' - \omega$
$K^- p \rightarrow K^- p$	Yes <sup>b</sup>	$P + P' + \omega$
$\bar{p} p \rightarrow \bar{p} p$	Yes <sup>c</sup>	$P + P' + \omega$

<sup>a</sup> Reference (a) of Table I.

<sup>b</sup> D. R. O. Morrison, Ref. 14.

<sup>c</sup> C. B. Chiu, Ref. 12; B. Musgrave, ANL Report No. ANL/HEP 6812, 1968 (unpublished).

Recently there have been other models accounting for the presence or absence of dipoles. We shall leave till Sec. V a comparison between these models and the one presented here.

## II. REACTIONS DOMINATED BY VECTOR AND TENSOR TRAJECTORIES

### A. Residues

The reactions listed in Table I are of the form meson ( $M_1$ ) + baryon ( $B_1$ )  $\rightarrow$  meson ( $M_2$ ) + baryon ( $B_2$ ),<sup>5</sup> where the dominant exchange is an odd-signature vector trajectory  $V$  or an even-signature tensor trajectory  $T$ . Assuming factorization of the residues of Regge poles, the amplitude for the first eight of the reactions of Table I is

$$\beta_{VM_1M_2} \beta_{VB_1B_2} \frac{1 - e^{-i\pi\alpha}}{\sin\pi\alpha} s^\alpha. \quad (1)$$

Basic considerations of the Regge-pole model give no compelling reason for the vanishing of the residue at  $t = -0.6 \text{ GeV}^2$  corresponding to  $\alpha = 0$ .<sup>6</sup> We appeal to duality and the existence of exotic channels to gain further information on these residues. The reaction  $M_1 + M_2 \rightarrow M_2 + M_1$  is dominated by the exchange of both a vector and a tensor trajectory. The residues of the tensor trajectories must vanish at  $\alpha = 0$ . The amplitude for this meson-meson reaction is

$$[(\beta_{VM_1M_2})^2 (1 - e^{-i\pi\alpha}) + (\beta_{TM_1M_2})^2 (1 + e^{-i\pi\alpha})] s^\alpha / \sin\pi\alpha, \quad (2)$$

with  $\beta_{TM_1M_2}$  vanishing as  $\alpha \rightarrow 0$ . If the channel  $M_1 + M_2$  or one related to it by isospin is exotic, then the imaginary part of Eq. (2) must vanish, yielding an equality of the residues of the even- and odd-signature trajectories, and consequently the appearance of a zero in  $\beta_{VM_1M_2}$  at  $\alpha = 0$ . If the channel  $M_1 + M_2$  is in no way related to an exotic one, no such argument can be given. Note that any time both  $M_1$  and  $M_2$  are isovectors,

<sup>5</sup> Photon-initiated reactions may be related to vector-meson-initiated processes.

<sup>6</sup> S. Mandelstam and L. Wang, Phys. Rev. **160**, 1490 (1967); C. E. Jones and V. L. Teplitz, *ibid.* **159**, 1271 (1967); J. H. Schwarz, *ibid.* **159**, 1269 (1967).

$M_1 + M_2$  may have isospin 2, which is exotic; if either is an isoscalar, there is no exotic channel related to  $M_1 + M_2$ .<sup>7</sup> Thus we find a rule that if both  $M_1$  and  $M_2$  are isovectors,  $\beta_{VM_1M_2}$  develops a zero as  $t$  approaches  $-0.6 \text{ GeV}^2$ . As the dominant Regge exchange vanishes at the above  $t$  value for these reactions, we expect them to exhibit a dip. We further note that this correlates perfectly with the presence or absence of dips in the first eight reactions of Table I.

We must still discuss the baryon-baryon-Regge-pole vertex. If we consider channels with baryon number two as exotic, then the above arguments may be applied to the reaction  $B_1 + B_2 \rightarrow B_2 + B_1$ , and we find that  $\beta_{VB_1B_2}$  vanishes as  $\alpha$  approaches zero. Thus the reactions of Table I split into two groups: (a) those whose residue develops a zero at both vertices and (b) those where a zero occurs only at one vertex. Let us tentatively assume that the residues for case (a) behave as  $\alpha^2$  and case (b) as  $\alpha$ .

In the next subsection we shall give a plausibility argument for why we expect a dip to occur only for case (a).

One could take the attitude that duality for baryon-baryon processes is not understood (as evidenced by the failure of duality diagrams for this case<sup>8</sup> and by the experimental observation that the total nucleon-nucleon cross section is not flat) and abandon the necessity for the vanishing of  $\beta_{VBB}$ . In this case it would still be the meson vertex which would determine what reactions exhibit a dip. However, the dip structure of  $B + B \rightarrow B + B$ , discussed below, favors the vanishing of  $\beta_{VBB}$ .<sup>9</sup>

### B. Possible Dip Mechanism

In this subsection we would like to present a possible mechanism which would permit a dip at a point corresponding to  $\alpha=0$  for a leading trajectory only for the case where the residue vanishes as  $\alpha^2$ , and no dip if the vanishing is only linear. We assume that  $\alpha$  is a linear function of  $t$  in the vicinity of its zero.

As mentioned in the Introduction, we consider all reactions to be dominated by the leading Regge exchanges except in the region where these exchanges vanish. Therefore we expect any other mechanisms, which we collectively label as background (b.g.), to be important in the dip region. This background may consist of lower-lying trajectories or, more likely, it is due to cuts associated with the leading trajectories. Let  $\beta\alpha^n$  denote the contribution of the Regge pole and b.g.

<sup>7</sup> Our assignment violates  $SU_3$  symmetry for residues of Regge trajectories. One possible way to reconcile it with  $SU_3$  is to assume that the  $\varphi$  trajectory (as opposed to the physical particle) couples to nucleons. This would imply that the  $\omega$ - $\varphi$  mixing angle varied along the trajectory.

<sup>8</sup> H. Harari, Phys. Rev. Letters 22, 562 (1969); J. Rosner, *ibid.* 22, 689 (1969).

<sup>9</sup> Factorization and the requirement that  $F/D$  be the same for the vector and tensor trajectories implies the vanishing of  $\beta_{VBB}$ ; see J. Rosner, C. Rebbi, and R. Slansky, Phys. Rev. 188, 2367 (1969).

the background term, which we assume not to vary rapidly in the region of interest. The cross section becomes

$$d\sigma = |\text{b.g.}|^2 + 2\beta\alpha^n \text{Re}(\text{b.g.}) + O(\alpha^{2n}). \quad (3)$$

Note that for  $n=1$  we would not observe a dip, while for  $n=2$  we would. In the remainder of this article, we shall assume that a dip will occur only if the residue vanishes quadratically in  $\alpha$ .

### C. Summary of Structure of Reaction of Table I

We have already discussed the structure of the first eight reactions of Table I in Sec. II A. For completeness, we have included the process  $\pi N \rightarrow \eta N$  and  $\pi N \rightarrow \eta \Delta$  dominated by the exchange of the single  $A_2$  trajectory. Without use of duality arguments, we expect the residue to behave as  $\alpha^2$ . Since we are dealing with an even-signature trajectory, one power of  $\alpha$  is canceled by the  $1/\sin\pi\alpha$  term. Employing the criteria of Sec. II B, we thus expect no dip, and none is observed.

## III. HELICITY STRUCTURE

Till now we have been rather cavalier about the spin complications on the residues. Before proceeding further, we wish to give a detailed discussion of these. Let us first consider the reaction  $\pi + \pi \rightarrow \pi + \pi$ . As mentioned in connection with Eq. (2),  $\beta_{\rho\pi\pi} = \beta_{\rho'\pi\pi}$ . We know that  $\beta_{\rho'\pi\pi}$  vanishes at  $\alpha=0$ , and the question is with which power it does so. The two possibilities are  $\beta_{\rho'\pi\pi} \sim \sqrt{\alpha}$  or  $\beta_{\rho'\pi\pi} \sim \alpha$ . Once a choice is made, then analyticity determines the nature of the zeros of all other reactions. The analyticity in question is the requirement that the full scattering amplitude have no branch point at  $t \sim -0.6 \text{ GeV}^2$ . We shall take as prototypes the reactions  $\pi^- p \rightarrow \pi^0 n$ ,  $\pi^- p \rightarrow \omega n$ , and  $N + N \rightarrow N + N$  (non-Pomeranchukon part). Considering the reaction  $\pi\pi \rightarrow \omega\pi$ , the first choice,  $\beta_{\rho'\pi\pi} \sim \sqrt{\alpha}$ , implies  $\beta_{\rho\pi\omega} \sim \sqrt{\alpha}$  ( $\text{const} \times \sqrt{\alpha}$  from Legendre polynomial).<sup>10</sup> For the second choice,  $\beta_{\rho'\pi\pi} \sim \alpha$ , we can have  $\beta_{\rho\pi\omega} \sim \text{const}$  ( $\sqrt{\alpha}$  from the Legendre polynomial  $\times 1/\sqrt{\alpha}$  from the Mandelstam-Wang fixed pole<sup>6</sup>). In Table III, we list the various possibilities for the three reactions mentioned above. Note that case 1 leads to the same behavior for  $\pi N \rightarrow \pi N$  as it does for  $\pi N \rightarrow \omega N$ , which is one thing we are trying to avoid. There are two further experimental results that favor case 2. If one assumes a nonflat Pomeranchukon trajectory, then the double zero in the elastic  $\pi^+ p$  and  $\pi^- p$  polarizations<sup>11</sup> requires a double zero in the non-Pomeranchukon part of the spin-flip amplitudes. Likewise, the dip structure of the elastic  $pp$  and  $\bar{p}p$  scattering requires a double zero in  $A_{NN}^{(S,S)}$ . We shall return to this point in the next section.

<sup>10</sup> L. Bertocchi, in *Proceedings of the International Conference on Elementary Particles, Heidelberg 1967*, edited by H. Filthuth (North-Holland, Amsterdam, 1968), p. 197.

<sup>11</sup> N. Booth, Rutherford Laboratory Report No. RPP/H/58, 1969 (unpublished).

TABLE III. Vertices and residues consistent with various assignments of the zero in  $\beta_{P'\pi\pi}$ . The superscripts  $S$  and  $N$  indicate whether the  $t$ -channel vertex is sense (no spin flip) or nonsense (spin flip).

Vertex	Case 1	Case 2
$\beta_{P'\pi\pi}$	$\sqrt{\alpha}$	$\alpha$
$\beta_{\rho\pi\pi}$	$\sqrt{\alpha}$	$\alpha$
$\beta_{\rho\pi\omega}$	$\sqrt{\alpha}$	const
$\beta_{P'NN}^{(S)} = \beta_{\rho NN}^{(S)}$	$\sqrt{\alpha}$	$\alpha$
$\beta_{P'NN}^{(N)} = \beta_{\rho NN}^{(N)}$	$\sqrt{\alpha}$	$\alpha$
Amplitude		
$A_{\pi N \rightarrow \pi N}^{(SS)}$	$\alpha$	$\alpha^2$
$A_{\pi N \rightarrow \pi N}^{(SN)}$	$\alpha$	$\alpha^2$
$A_{\pi N \rightarrow \omega N}^{(NS)}$	$\alpha$	$\alpha$
$A_{\pi N \rightarrow \omega N}^{(NN)}$	$\alpha$	$\alpha$
$A_{NN \rightarrow NN}^{(SS)}$	$\alpha$	$\alpha^2$
$A_{NN \rightarrow NN}^{(SN)}$	$\alpha$	$\alpha^2$
$A_{NN \rightarrow NN}^{(NN)}$	$\alpha$	$\alpha^2$

#### IV. REACTIONS DOMINATED BY SEVERAL TRAJECTORIES

A list of reactions dominated by several trajectories is presented in Table II. We grouped these into three categories: (i) no Pomeranchukon contribution; (ii) elastic reactions with exotic  $s$  channels; and (iii) line-reversed reactions of the above. In the first case the situation is similar to the last two reactions of Table I, where we expect no dip due to the presence of tensor as well as vector trajectories. To account for the structure of the other reactions, we shall take the attitude that even-signature trajectories contain a factor  $\alpha$  at each vertex and thus the residue vanishes as  $\alpha^2$ ; by duality arguments the same holds true for the odd-signature trajectories. The amplitude for exotic  $s$ -channel reactions ( $p\bar{p}$ ,  $n\bar{p}$ ,  $K^+p$ ) is

$$isf(t) - C\alpha^2 s^\alpha / \sin\pi\alpha, \quad (4)$$

with  $C$  real, positive, and slowly varying in  $t$ . The amplitude for the time-reversed nonexotic  $s$  channels ( $\bar{p}p$ ,  $\bar{n}p$ ,  $K^-p$ ) is

$$isf(t) - C\alpha^2 e^{-i\pi\alpha} s^\alpha / \sin\pi\alpha. \quad (5)$$

In the first case, as  $\alpha$  goes to zero, the cross section is  $|f(t)|^2 + O(1/s^2)$ , and we do not predict a dip. The cross section in the second case has a contribution from an interference term and is

$$d\sigma/dt \sim |f(t)|^2 + 2C\alpha^2 f(t)/s + O(1/s^2). \quad (6)$$

We expect a dip near  $t = -0.6 \text{ GeV}^2$  whose exact position varies with energy. This variation of dip position has been observed experimentally.<sup>12</sup> Had we taken only one power of  $\alpha$  in the residue, we would not have been able to obtain this dip in Eq. (6). It is interesting to note that this  $\alpha^2$  parametrization of the non-Pomeranchukon

trajectories was used in a previous phenomenological fit to  $\bar{p}p$  scattering.<sup>13</sup>

#### V. CONCLUSION

Using duality arguments, we have been able to account for the dip structure of a large number of two-body reactions. The consistency of these arguments, coupled with a study of the spin structure of various reactions, indicated that reactions exhibiting a dip at  $t$  corresponding to  $\alpha=0$  vanish as  $\alpha^2$ . We believe that this assignment is consistent with other experimental data as  $NN$  scattering and polarization in  $\pi p$  elastic scattering.

These arguments may be extended to processes dominated by the exchange of  $K^*$  and  $K^{**}$  trajectories. As these reactions fall into the same category as those dominated by both the  $\rho$  and  $A_2$ , we do not expect any dips to occur. Experimentally, there is no evidence for dips in these reactions which would occur at  $t \sim -0.3 \text{ GeV}^2$ .<sup>14</sup> We are incapable of extending this model to backward directions involving baryon exchange. A naive extension would predict no dip in backward  $\pi N$  scattering, which, however, exhibits a clear dip at  $u = -0.15 \text{ GeV}^2$ , corresponding to a nonsense point in the  $N_\alpha$  trajectory.

Recently, other groups have suggested explanations for the dip structure in the reactions considered here. These do not use duality arguments at all, but either use cuts<sup>15</sup> in addition to poles or rely on the strength of the fixed Mandelstam-Wang poles.<sup>16</sup> All of these models rely heavily on the spin structure of these reactions. It is amazing that for the reactions considered by any of the models, including ours, the agreement with experiment, for the cases where good experimental data are available, is perfect. We conclude this comparison with one reaction which may serve to discriminate among these models. It is

$$\pi^+ p \rightarrow A_2^0 \Delta^{++}. \quad (7)$$

Using our criterion, we would expect a dip, while some of the other models do not predict one. At present the experimental situation is unclear<sup>17</sup>; however, we feel there is some indication of a dip structure.

#### ACKNOWLEDGMENTS

We wish to thank Dr. Haim Harari for calling our attention to this problem and for a discussion of his work. Useful discussions were also had with Dr. Syd Meshkov, Dr. C. Chiu, and Dr. S. Matsuda.

<sup>13</sup> C. B. Chiu, S. Y. Chu, and L. L. Wang, Phys. Rev. **161**, 1563 (1967).

<sup>14</sup> D. R. O. Morrison, in Proceedings of the Lund International Conference on Elementary Particles, Lund, 1969 (unpublished).

<sup>15</sup> H. Harari, Ref. 4; F. Henyey, G. L. Kane, J. Pumplin, and M. H. Ross, Phys. Rev. **182**, 1579 (1969); A. Dar, DESY report (unpublished); R. Carlitz and M. Kislinger, Phys. Rev. **D 2**, 336 (1970).

<sup>16</sup> C. B. Chiu and S. Matsuda, Cal. Tech. Report, 1970 (unpublished).

<sup>17</sup> Aachen-Berlin-CERN Collaboration, Nucl. Phys. **B8**, 45 (1968).

<sup>12</sup> C. B. Chiu, Rev. Mod. Phys. **41**, 640 (1969).